kneed

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Contents

This is the documentation for the kneed Python package. Given *x* and *y* arrays, *kneed* attempts to identify the knee/elbow point of a line fit to the data. The knee/elbow is defined as the point of the line with maximum curvature. For more information about how each of the parameters affect identification of knee points, check out *Parameter Examples*. For a full reference of the API, head over to the *API Reference*.

CHAPTER 1

Parameter Examples

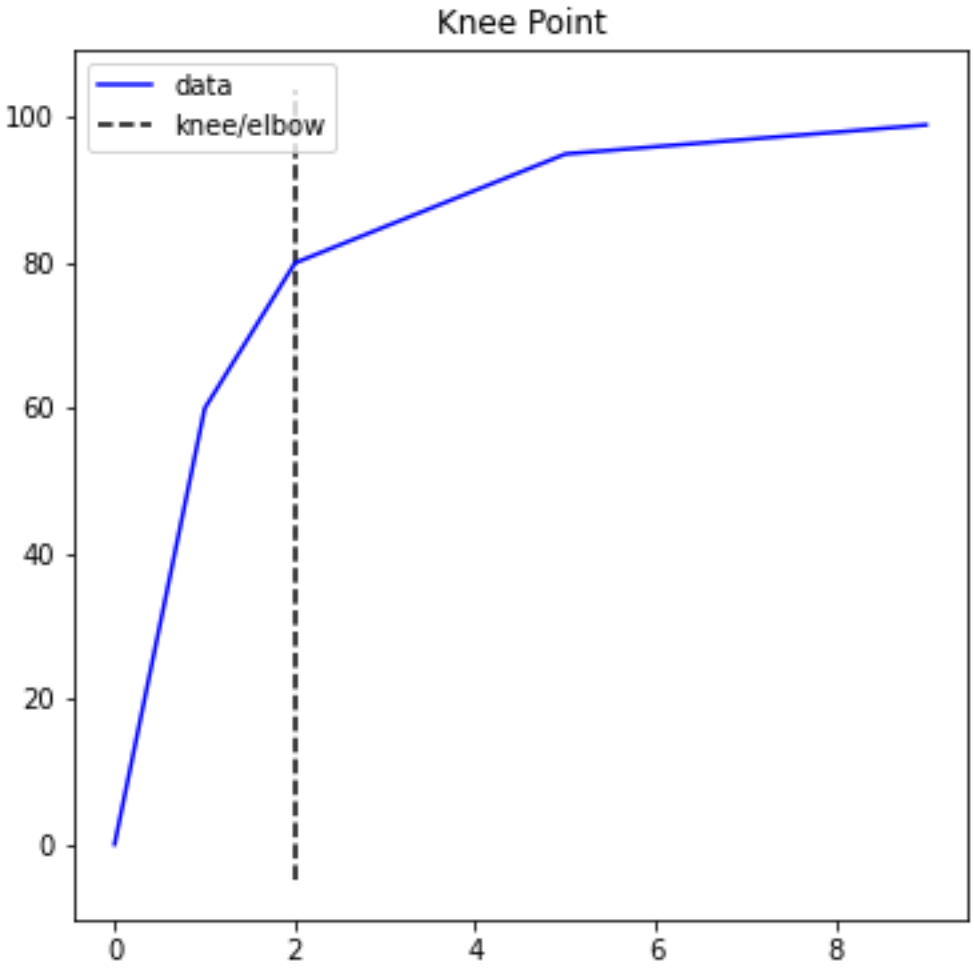
This page provides examples that outline the effect of tuning the parameters for *KneeLocator*.

1.1 curve

If *curve="concave"*, kneed will detect knees. If *curve="convex"*, it will detect elbows. Use the *DataGenerator* class to generate synthetic data:

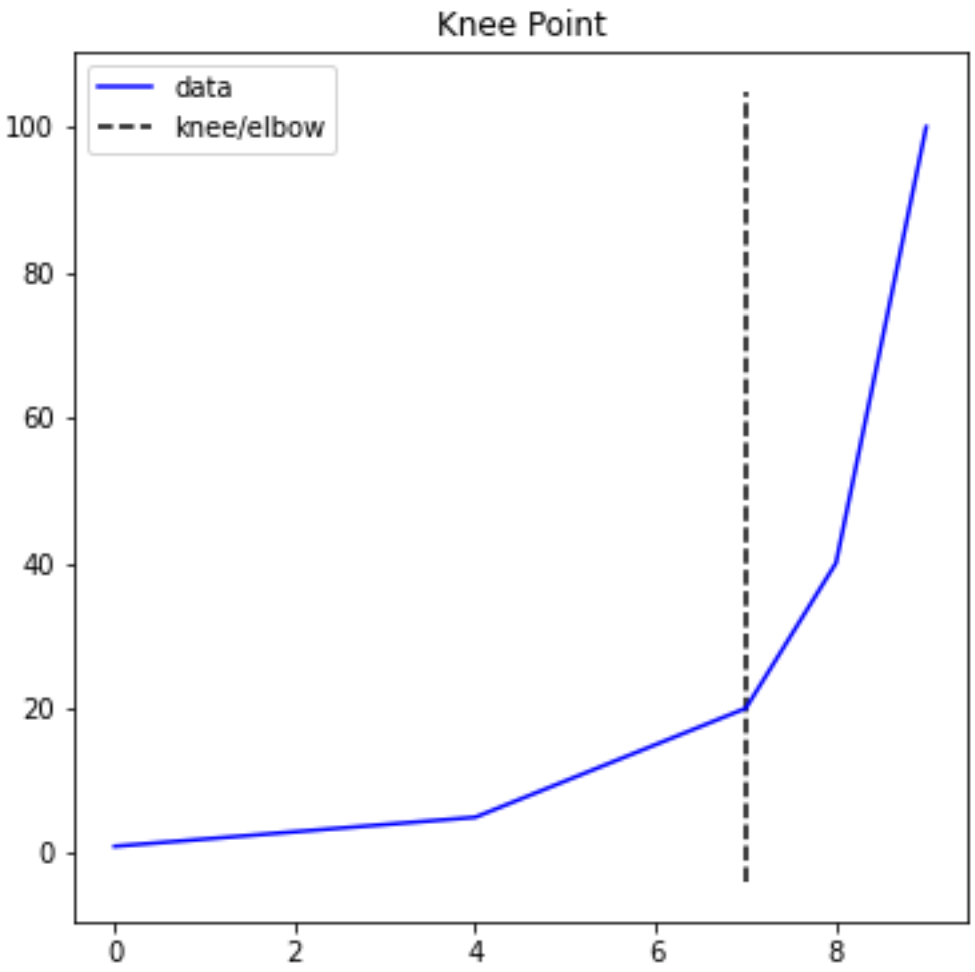
An concave curve example:

```
from kneed import KneeLocator, DataGenerator as dg
x, y = dg.concave_increasing()
kl = KneeLocator(x, y, curve="concave")
kl.plot_knee()
```



A convex curve example:

```
from kneed import KneeLocator, DataGenerator as dg
x, y = dg.convex_increasing()
kl = KneeLocator(x, y, curve="convex")
kl.plot_knee()
```

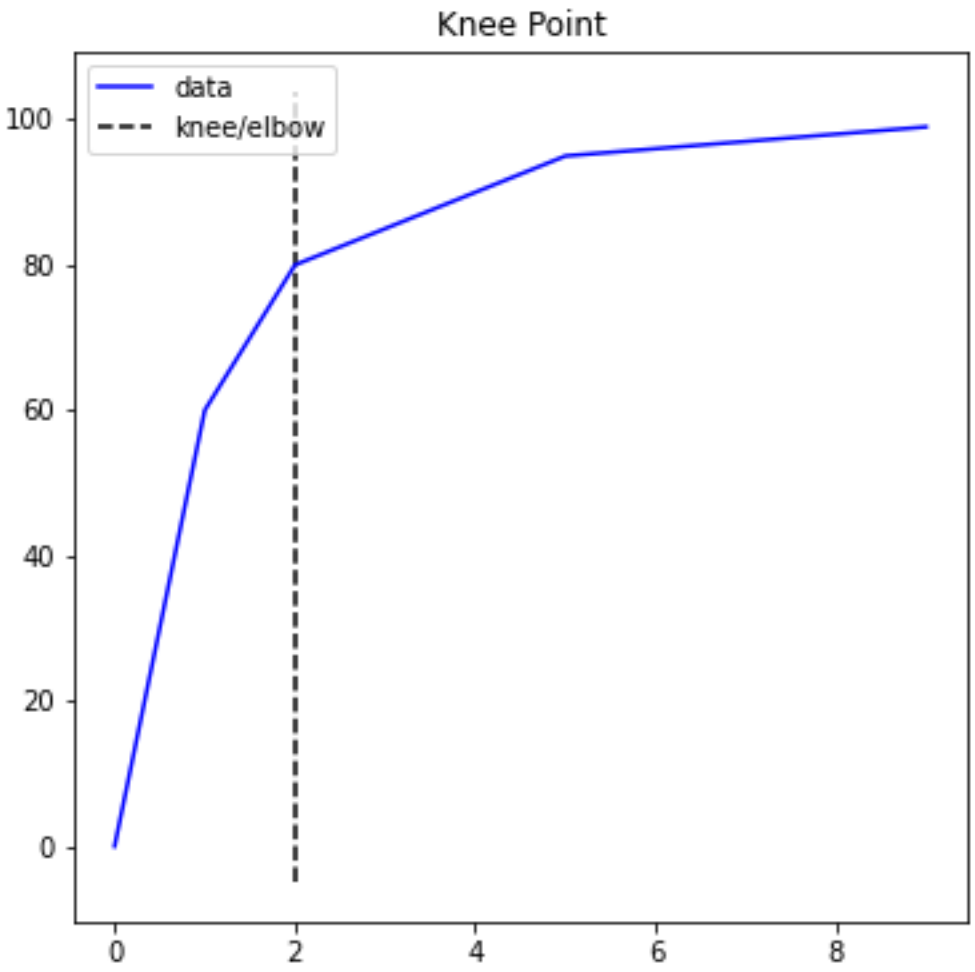


1.2 direction

The direction parameter describes the line from left to right on the x-axis. If the knee/elbow you are trying to identify is on a positive slope use *direction="increasing"*, if the knee/elbow you are trying to identify is on a negative slope, use *direction="decreasing"*. Use the *DataGenerator* class to generate synthetic data.

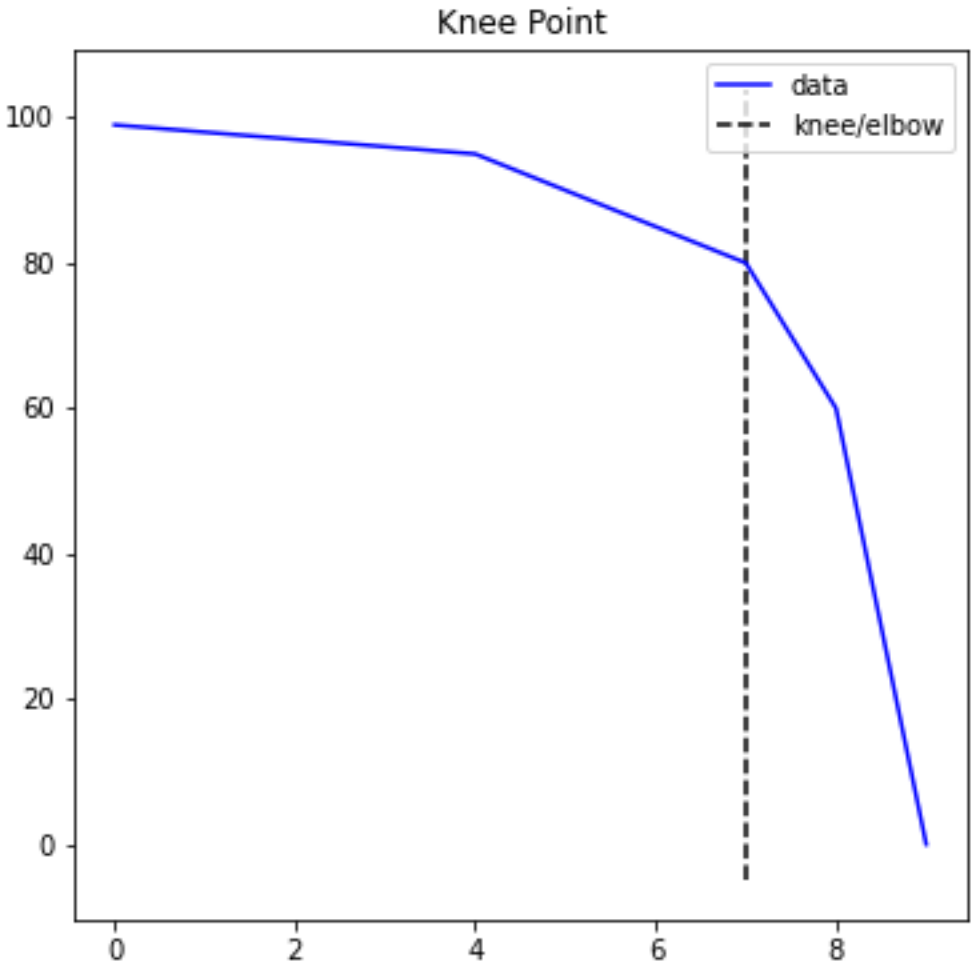
An example of an increasing curve:

```
from kneed import KneeLocator, DataGenerator as dg
x, y = dg.concave_increasing()
kl = KneeLocator(x, y, curve="concave", direction="increasing")
kl.plot_knee()
```



An example of a decreasing curve:

```
from kneed import KneeLocator, DataGenerator as dg
x, y = dg.concave_decreasing()
kl = KneeLocator(x, y, curve="concave", direction="decreasing")
kl.plot_knee()
```



1.3 S

The selected knee point is tunable by setting the sensitivity parameter *S*. From the kneedle manuscript:

> The sensitivity parameter allows us to adjust how aggressive we want Kneedle to be when detecting knees. Smaller values for S detect knees quicker, while larger values are more conservative. Put simply, S is a measure of how many "flat" points we expect to see in the unmodified data curve before declaring a knee.

```
import numpy as np

np.random.seed(23)

sensitivity = [1, 3, 5, 10, 100, 200, 400]
knees = []
norm_knees = []
```

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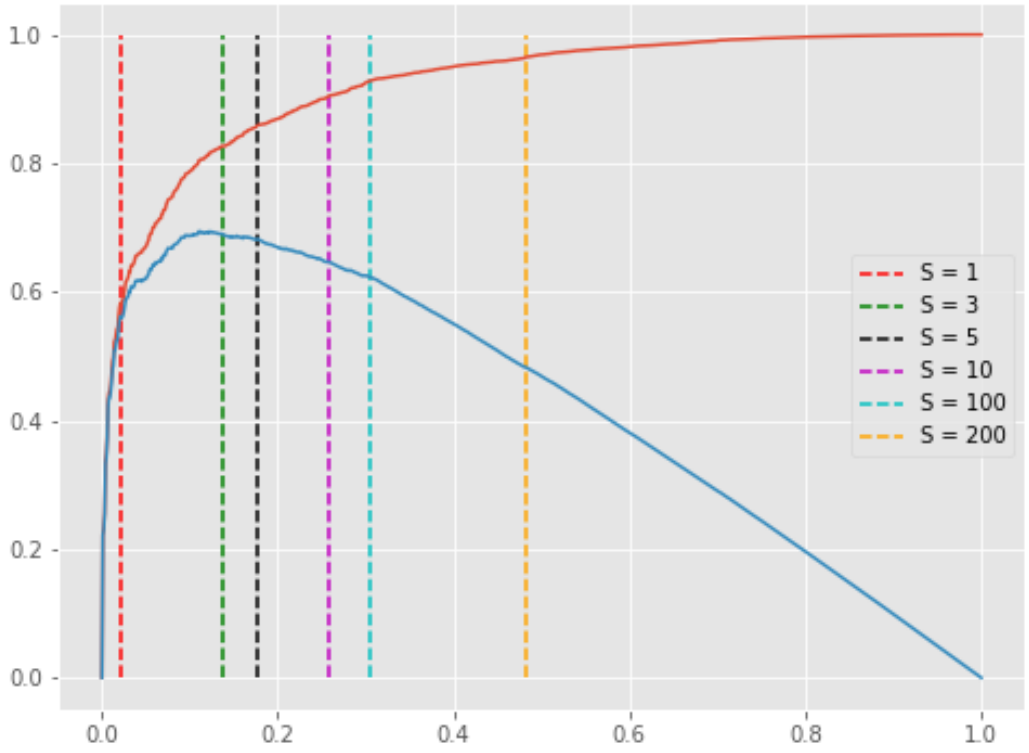
```
n = 1000
x = range(1, n + 1)
y = sorted(np.random.gamma(0.5, 1.0, n), reverse=True)
for s in sensitivity:
    kl = KneeLocator(x, y, curve="convex", direction="decreasing", S=s)
    knees.append(kl.knee)
    norm_knees.append(kl.norm_knee)

print(knees)
[43, 137, 178, 258, 305, 482, 482]

print([nk.round(2) for nk in norm_knees])
[0.04, 0.14, 0.18, 0.26, 0.3, 0.48, 0.48]

import matplotlib.pyplot as plt

plt.style.use("ggplot")
plt.figure(figsize=(8, 6))
plt.plot(kl.x_normalized, kl.y_normalized)
plt.plot(kl.x_difference, kl.y_difference)
colors = ["r", "g", "k", "m", "c", "orange"]
for k, c, s in zip(norm_knees, colors, sensitivity):
    plt.vlines(k, 0, 1, linestyles="--", colors=c, label=f"S = {s}")
plt.legend()
```



Any *S*>200 will result in a knee at 482 (0.48, normalized) in the plot above.

1.4 online

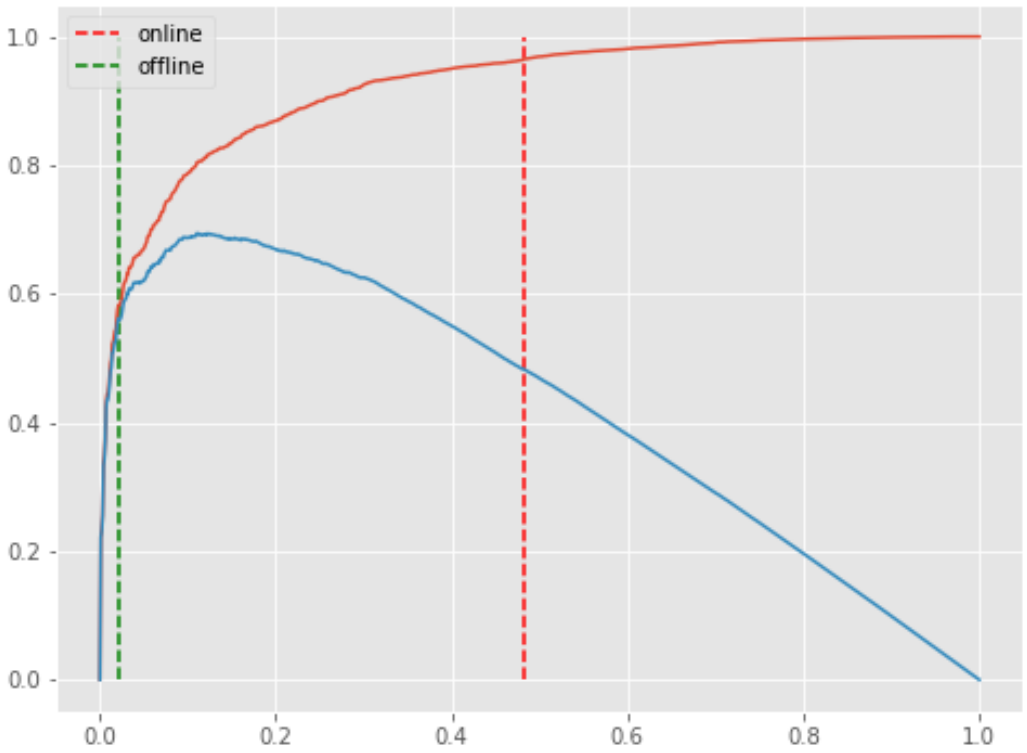
The knee point can be corrected if the parameter online is *True* (default). This mode will step through each element in x. In contrast, if online is False, kneed will run in offline mode and return the first knee point identified. When *online=False* the first knee point identified is returned regardless of whether it's the local maxima on the difference curve or the global maxima. So the algorithm stops early. When *online=True*, kneed runs in online mode and "corrects" itself by continuing to overwrite any previously identified knees.

Using the *x* and *y* from the sensitivity example above, this time, keep *S=1* but modify *online*.

```
kl_online = KneeLocator(x, y, curve="convex", direction="decreasing", online=True)
kl_offline = KneeLocator(x, y, curve="convex", direction="decreasing", online=False)

import matplotlib.pyplot as plt

plt.style.use("ggplot")
plt.figure(figsize=(8, 6))
plt.plot(kl_online.x_normalized, kl_online.y_normalized)
plt.plot(kl_online.x_difference, kl_online.y_difference)
colors = ["r", "g"]
for k, c, o in zip(
    [kl_online.norm_knee, kl_offline.norm_knee], ["r", "g"], ["online", "offline"]
):
    plt.vlines(k, 0, 1, linestyles="--", colors=c, label=o)
plt.legend()
```



1.5 interp_method

This parameter controls the interpolation method for fitting a spline to the input *x* and *y* data points. Valid arguments are *"interp1d"* and *"polynomial"*.

If *interp_method="interp1d"*, then *x* and *y* will be fit using scipy.interpolate.interp1d.

```
x = list(range(90))
y = [
    7304, 6978, 6666, 6463, 6326, 6048, 6032, 5762, 5742,
    5398, 5256, 5226, 5001, 4941, 4854, 4734, 4558, 4491,
    4411, 4333, 4234, 4139, 4056, 4022, 3867, 3808, 3745,
    3692, 3645, 3618, 3574, 3504, 3452, 3401, 3382, 3340,
    3301, 3247, 3190, 3179, 3154, 3089, 3045, 2988, 2993,
    2941, 2875, 2866, 2834, 2785, 2759, 2763, 2720, 2660,
    2690, 2635, 2632, 2574, 2555, 2545, 2513, 2491, 2496,
    2466, 2442, 2420, 2381, 2388, 2340, 2335, 2318, 2319,
    2308, 2262, 2235, 2259, 2221, 2202, 2184, 2170, 2160,
    2127, 2134, 2101, 2101, 2066, 2074, 2063, 2048, 2031
]

kneedle = KneeLocator(
    x, y, S=1.0, curve="convex", direction="decreasing", interp_method="interp1d"
```

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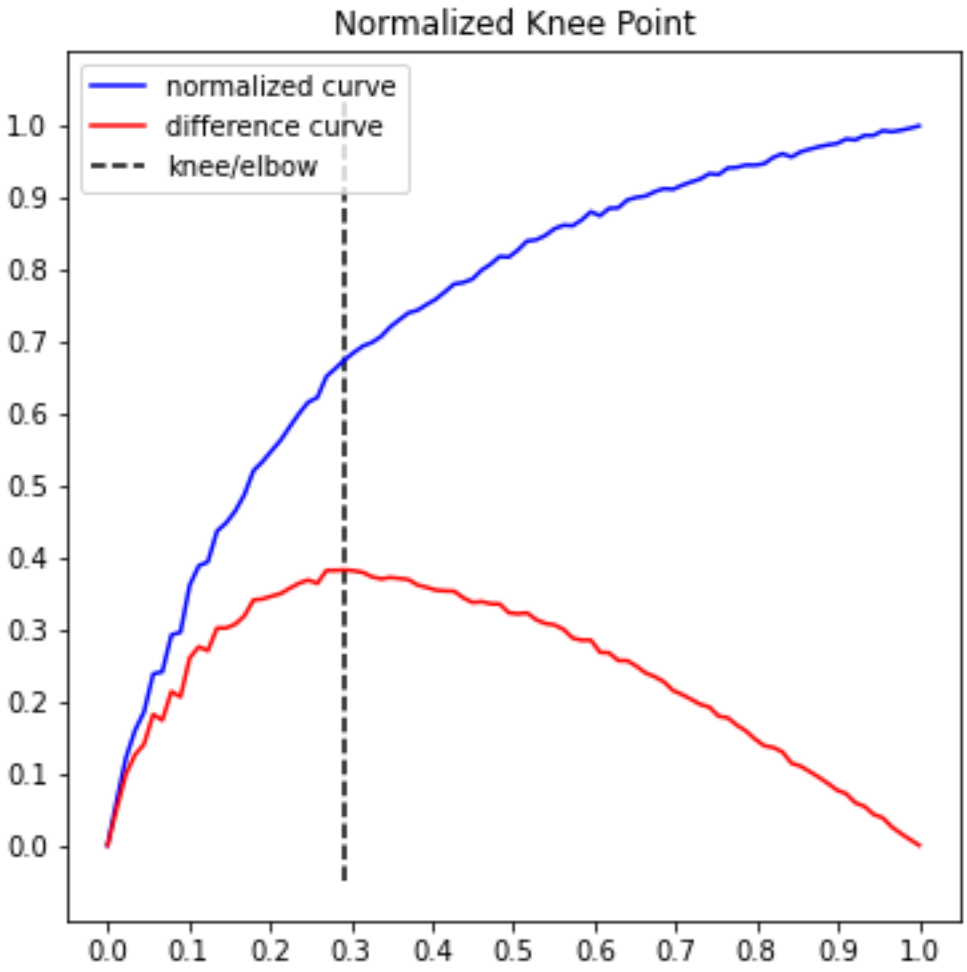
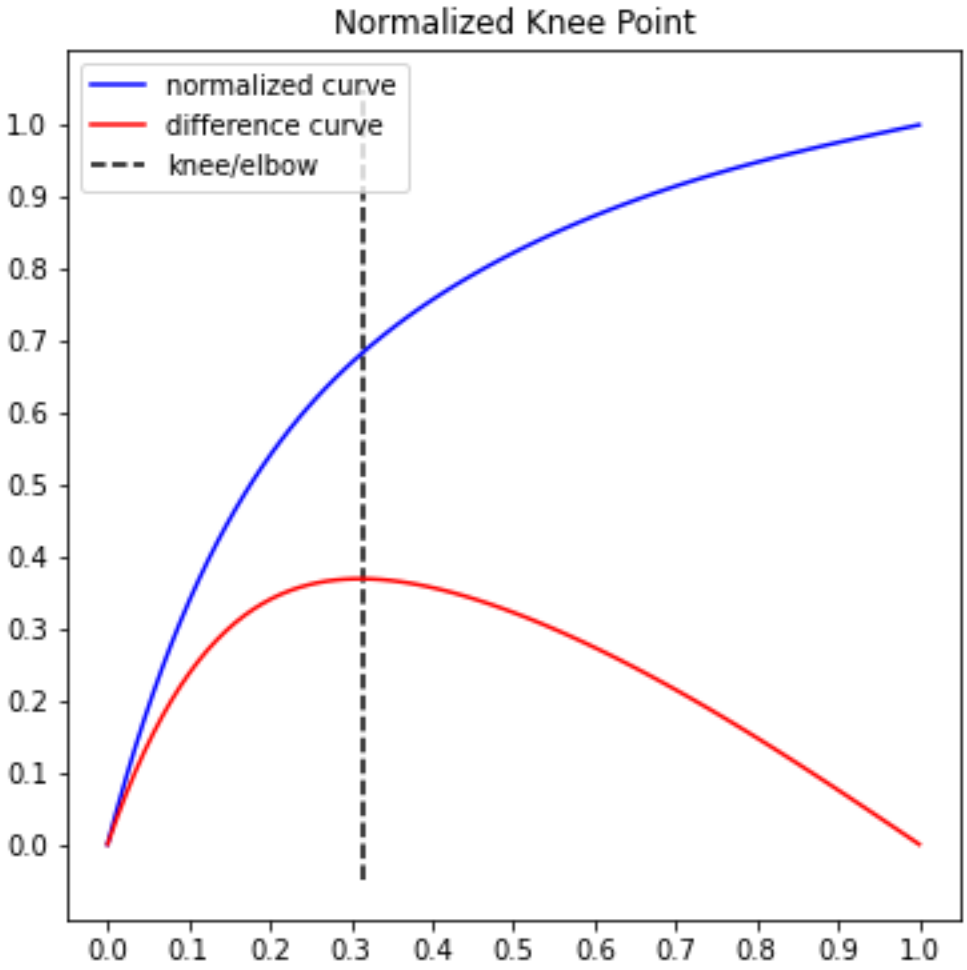
```
)
kneedle.plot_knee_normalized()
```



If *interp_method="polynomial"*, then *x* and *y* will be fit using numpy.polyfit. Using the same data, change *interp_method* and note that the line is smoother.

```
kneedle = KneeLocator(
    x, y, S=1.0, curve="convex", direction="decreasing", interp_method="polynomial",
)
kneedle.plot_knee_normalized()
```

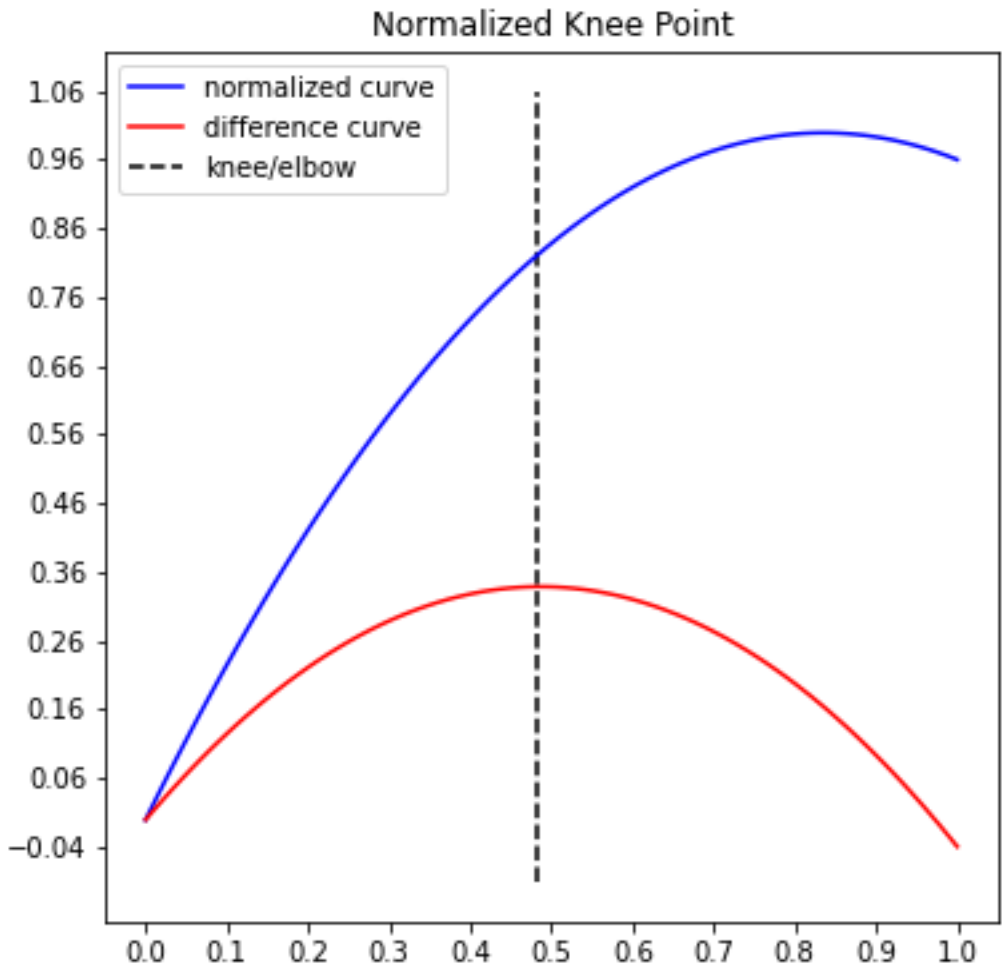


1.6 polynomial_degree

This parameter controls the degree of the polynomial fit. This parameter is passed as the argument to the *deg* parameter in numpy.polyfit.

Using the same data from the *interp_method* example, note how the line (and knee point) change when *polynomial_degree=2* instead of the default value, *7*:

```
kneedle = KneeLocator(
    x, y, S=1.0, curve="convex", direction="decreasing", interp_method="polynomial",
    polynomial_degree=2
)
kneedle.plot_knee_normalized()
```


Normalized Knee Point
normalized curve
difference curve
knee/elbow
1.06
0.96
0.86
0.76
0.66
0.56
0.46
0.36
0.26
0.16
0.06
−0.04
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

CHAPTER 2

API Reference

There are two classes in *kneed*: *KneeLocator* identifies the knee/elbow point(s) and and *DataGenerator* creates synthetic *x* and *y* numpy arrays to explore *kneed*.

2.1 KneeLocator

class kneed.knee_locator.**KneeLocator**(*x: Iterable[float], y: Iterable[float], S: float = 1.0, curve: str = 'concave', direction: str = 'increasing', interp_method: str = 'interp1d', online: bool = False, polynomial_degree: int = 7*)

Once instantiated, this class attempts to find the point of maximum curvature on a line. The knee is accessible via the *.knee* attribute.

Parameters

- **x** (*array-like*) – x values.
- **y** (*array-like*) – y values.
- **S** (*float*) – Sensitivity, original paper suggests default of 1.0
- **curve** (*str*) – If 'concave', algorithm will detect knees. If 'convex', it will detect elbows.
- **direction** (*str*) – one of {"increasing", "decreasing"}
- **interp_method** (*str*) – one of {"interp1d", "polynomial"}
- **online** (*bool*) – kneed will correct old knee points if True, will return first knee if False
- **polynomial_degree** (*int*) – The degree of the fitting polynomial. Only used when interp_method="polynomial". This argument is passed to numpy polyfit *deg* parameter.

Variables

- **x** (*array-like*) – x values.
- **y** (*array-like*) – y values.

- **S** (*integer*) – Sensitivity, original paper suggests default of 1.0
- **curve** (*str*) – If 'concave', algorithm will detect knees. If 'convex', it will detect elbows.
- **direction** (*str*) – one of {"increasing", "decreasing"}
- **interp_method** (*str*) – one of {"interp1d", "polynomial"}
- **online** (*str*) – kneed will correct old knee points if True, will return first knee if False
- **polynomial_degree** (*int*) – The degree of the fitting polynomial. Only used when interp_method="polynomial". This argument is passed to numpy polyfit *deg* parameter.
- **N** (*integer*) – The number of *x* values in the
- **all_knees** (*set*) – A set containing all the x values of the identified knee points.
- **all_norm_knees** (*set*) – A set containing all the normalized x values of the identified knee points.
- **all_knees_y** (*list*) – A list containing all the y values of the identified knee points.
- **all_norm_knees_y** (*list*) – A list containing all the normalized y values of the identified knee points.
- **Ds_y** (*numpy array*) – The y values from the fitted spline.
- **x_normalized** (*numpy array*) – The normalized x values.
- **y_normalized** (*numpy array*) – The normalized y values.
- **x_difference** (*numpy array*) – The x values of the difference curve.
- **y_difference** (*numpy array*) – The y values of the difference curve.
- **maxima_indices** (*numpy array*) – The indices of each of the maxima on the difference curve.
- **maxima_indices** – The indices of each of the maxima on the difference curve.
- **x_difference_maxima** (*numpy array*) – The x values from the difference curve where the local maxima are located.
- **y_difference_maxima** (*numpy array*) – The y values from the difference curve where the local maxima are located.
- **minima_indices** (*numpy array*) – The indices of each of the minima on the difference curve.
- **minima_indices** – The indices of each of the minima on the difference curve.
- **x_difference_minima** (*numpy array*) – The x values from the difference curve where the local minima are located.
- **y_difference_minima** (*numpy array*) – The y values from the difference curve where the local minima are located.
- **Tmx** (*numpy array*) – The y values that correspond to the thresholds on the difference curve for determining the knee point.
- **knee** (*float*) – The x value of the knee point.
- **knee_y** (*float*) – The y value of the knee point.
- **norm_knee** (*float*) – The normalized x value of the knee point.
- **norm_knee_y** (*float*) – The normalized y value of the knee point.

- **all_knees** – The x values of all the identified knee points.
- **all_knees_y** – The y values of all the identified knee points.
- **all_norm_knees** – The normalized x values of all the identified knee points.
- **all_norm_knees_y** – The normalized y values of all the identified knee points.
- **elbow** (*float*) – The x value of the elbow point (elbow and knee are interchangeable).
- **elbow_y** (*float*) – The y value of the knee point (elbow and knee are interchangeable).
- **norm_elbow** – The normalized x value of the knee point (elbow and knee are interchangeable).
- **norm_elbow_y** (*float*) – The normalized y value of the knee point (elbow and knee are interchangeable).
- **all_elbows** (*set*) – The x values of all the identified knee points (elbow and knee are interchangeable).
- **all_elbows_y** – The y values of all the identified knee points (elbow and knee are interchangeable).
- **all_norm_elbows** (*set*) – The normalized x values of all the identified knee points (elbow and knee are interchangeable).
- **all_norm_elbowss_y** – The normalized y values of all the identified knee points (elbow and knee are interchangeable).

2.1.1 Plotting methods

There are two methods for basic visualizations of the knee/elbow point(s).

KneeLocator.**plot_knee**(*figsize: Optional[Tuple[int, int]] = None*)
Plot the curve and the knee, if it exists

Parameters **figsize** – Optional[Tuple[int, int] The figure size of the plot. Example (12, 8)

Returns NoReturn

KneeLocator.**plot_knee_normalized**(*figsize: Optional[Tuple[int, int]] = None*)
Plot the normalized curve, the difference curve (x_difference, y_normalized) and the knee, if it exists.

Parameters **figsize** – Optional[Tuple[int, int] The figure size of the plot. Example (12, 8)

Returns NoReturn

2.2 DataGenerator

class kneed.data_generator.**DataGenerator**
Generate synthetic data to work with kneed.

static bumpy() → Tuple[Iterable[float], Iterable[float]]
Generate a sample function with local minima/maxima.

Returns tuple(x, y)

static concave_decreasing() → Tuple[Iterable[float], Iterable[float]]
Generate a sample decreasing concave function.

Returns tuple(x, y)

static concave_increasing() → Tuple[Iterable[float], Iterable[float]]
Generate a sample increasing concave function.

Returns tuple(x, y)

static convex_decreasing() → Tuple[Iterable[float], Iterable[float]]
Generate a sample decreasing convex function.

Returns tuple(x, y)

static convex_increasing() → Tuple[Iterable[float], Iterable[float]]
Generate a sample increasing convex function.

Returns tuple(x, y)

static figure2() → Tuple[Iterable[float], Iterable[float]]
Recreate the values in figure 2 from the original kneedle paper.

Returns tuple(x, y)

static noisy_gaussian(*mu: float = 50*, *sigma: float = 10*, *N: int = 100*, *seed=42*) → Tuple[Iterable[float], Iterable[float]]
Recreate NoisyGaussian from the orignial kneedle paper.

Parameters

- **mu** – The mean value to build a normal distribution around
- **sigma** – The standard deviation of the distribution.
- **N** – The number of samples to draw from to build the normal distribution.
- **seed** – An integer to set the random seed.

Returns tuple(x, y)

CHAPTER 3

Indices and tables

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